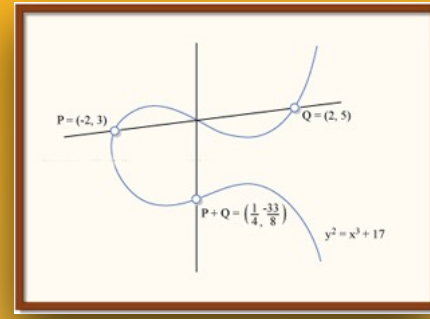


# Number Theory and Combinatorics II

November 1, 2024

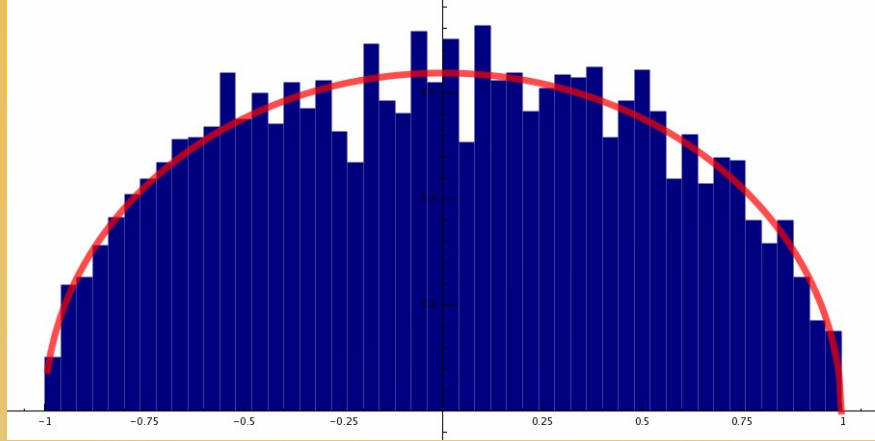


## Invited Speakers

Ayhan Günaydın *Boğaziçi University*

Özlem Ejder *Koç University*

Faruk Temur *İzmir Institute of Technology*



Place: IZTECH, Mathematics Department, Seminar room 122  
Time : 09:45-16:45

Registration deadline: October 25, 2024  
For registrations visit : <https://math.iyte.edu.tr>

Organizers: Haydar Göral  
Sadık Eyidoğan  
Sinem Benli Göral  
Mustafa Kutay Kutlu



NUMBER THEORY AND  
COMBINATORICS WORKSHOP II

ABSTRACT BOOK

İZMİR INSTITUTE OF TECHNOLOGY  
NOVEMBER 1, 2024

Dear Participants,

Number Theory and Combinatorics II is a daily event, whose goal is to bring together the people working on topics related to number theory and combinatorics. The first Number Theory and Combinatorics workshop was on November 18, 2022, and it took place at the mathematics department of İzmir Institute of Technology. The first Number Theory and Combinatorics workshop was supported by the Turkish Mathematical Society. Our current workshop, Number Theory and Combinatorics II, is also supported by the Turkish Mathematical Society. We are very grateful and indebted to them once more. We also would like to thank all researchers who will contribute either by their talks or their participation. We believe this event will be a good opportunity for our participants to learn more about number theory and combinatorics. As a final word, welcome and enjoy!

Yours Sincerely, Organizing Committee

### ORGANIZING COMMITTEE

Haydar Göral  
Sadık Eyidođan  
Sinem Benli Göral  
Mustafa Kutay Kutlu

*İzmir Institute of Technology*  
*Çukurova University*  
*İzmir Institute of Technology*  
*İzmir Institute of Technology*

## PROGRAM

09:45–10:15 : Opening

10:15–11:30 : Ayhan Günaydın

11:30–13:30 : Lunch Break

13:30–14:45 : Özlem Ejder

14:45–15:15 : Coffee Break

15:15–16:30 : Faruk Temur

16:30–16:45 : Closing

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# Beatty Sequences

Ayhan Günaydın

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The Beatty sequence generated by an irrational  $r > 1$  is  $([nr] : n > 0)$ , where  $[c]$  denotes the integer part of a real number  $c$ . A well-known property of this sequence is that "any pattern that appears once has to appear infinitely often; moreover we may determine when that pattern appears next time with a small error". After explaining what this means along with some other properties enjoyed by Beatty sequences, we will present the proof of a strengthening of it. Our proof depends on the model theoretic study of such sequences and all the necessary background will be overviewed. (Parts of this work is done joint with Melissa Özsahakyan).

# Groups, Geometry, Arithmetic and Dynamics

Özlem Ejder

Koç University  
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Let  $a_0$  be an integer, let  $f$  be a polynomial, and consider the sequence  $a_n = f(a_{n-1})$ . It is a natural question to ask whether there are infinitely many primes in this sequence. One quickly decides that there are not enough tools at hand to deal with this question and one asks instead about the primes dividing at least one term of the sequence. It turns out that the symmetries of the pre-images of  $a_0$  under the iterates of  $f$  play an essential role in the solution of this density question. Motivated by the prime density questions, we study the Galois theory of the iterates of a polynomial (or a rational function). We see fruitful connections between geometry, dynamics, arithmetic, and group theory in this subject. Some of the results presented in this talk are joint work with I. Bouw, V. Karemaker, Y. Kara, E. Ozman. This work is now supported by TÜBİTAK project 124F203.

# Exponential Sums Randomized with Stochastic Processes and Lattice Points in Regions

Faruk Temur

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In this talk we will explore random exponential sums of the form

$$\left\| \sum_{j \in A} a_j e^{2\pi i X(j)y} \right\|_{L^p(\mathbb{T})}^p$$

where  $A$  is a finite subset of integers, and  $X$  is a stochastic process. We will use these sums to investigate two esteemed problems of harmonic analysis and analytic number theory, the Hardy-Littlewood majorant problem, and  $L^{2n}(\mathbb{T})$ ,  $n \in \mathbb{N}$  norms of exponential sums. We will relate these sums for  $p = 2n$  and  $a_j = 0, 1$  to problems on lattice points in regions such as the Dirichlet divisor problem and the Gauss circle problem. The regions that we will encounter have the form of shells around hypersurfaces. This can be seen as an extension of the interpretation of  $L^{2n}(\mathbb{T})$ ,  $n \in \mathbb{N}$  norms of deterministic exponential sums as counting lattice points on surfaces. This is joint work with Cihan Sahilliođulları.