Ayın Sorusu

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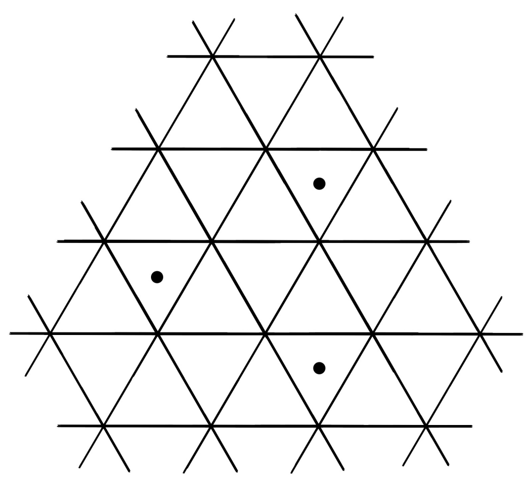
Soru: Düzlemde her biri 5 doğru parçasından oluşan 3 paralel çizgi grubu Şekil 1'deki gibi kesişmektedir. Kesişimler arasında kalan üçgenler eşkenardır. Şeklin 120 ve 240 derecelik rotasyonları sonucu birbiriyle yer değiştiren üçgenlere denk üçgen grubu diyelim. Örnek olarak Şekil 1'de içlerine nokta koyulmuş üçgenler bir denk üçgen grubudur. Her bir üçgenin içine 0 veya 1 yazarak çeşitli dizilişler oluşturduğumuzda, her iki paralel çizgi arasında ve her denk üçgen grubu içinde kalan sayıların toplamı çift olan x kadar diziliş varsa, her iki paralel çizgi arasında kalan sayıların toplamı çift olan kaç diziliş vardır?



Şekil 1

Solution:

Let the structure in Figure 1 represent a sequence in each case. First of all, let's define the sequence groups mentioned in the question as clusters.

$D =$ sequences in which the sums of the numbers between parallel lines are even,
$S =$ arrangements below D where the numbers in equilateral triangles are the same,
$K =$ arrays in which the sum of the numbers in equilateral triangles is even below D.

Now let's define a map from the set $S \times K$ to D:

$$\begin{aligned} \phi : S \times K &\to D, \\ (A, B) &\to A + B, \end{aligned} \tag{1}$$

and show that the ϕ transformation defined by the above operation is one-to-one and onto.

One-to-one:
We need to show that $\phi(A, B) = \phi(C, D) \Rightarrow (A, B) = (C, D)$

$$\begin{aligned} \phi(A, B) = \phi(C, D) &\Rightarrow A + B = C + D, \\ &\Rightarrow A - C = B - D. \end{aligned}$$

So $A - C \in S$ and $B - D \in K$. We can say $A - C \in S \cap K$ and $B - D \in S \cap K$. Here we have $S \cap K = \{\theta\}$ and $\{\theta\}$ represents the sequence where the number inside each triangle is 0. To see this, if a sequence is indeed in S , the numbers inside equilateral triangles must be the same. Since this sequence is also in K, the sum of the numbers in equilateral triangles must be even. The numbers in equilateral triangles in D must be 0. So,

$$\begin{aligned} A - C = D - B &= \theta, \\ (A, B) &= (C, D). \end{aligned}$$

Onto:

Let's take $M \in D$. Are there elements A and B with $A \in S$ and $B \in K$ such that $A + B = M$?

To see this, let's take M_{120} and M_{240} as 120 degree and 240 degree rotations of M, respectively.

$$M = M + M_{120} + M_{240} - M_{120} - M_{240},$$

and since

$$M_{120} \cong -M_{120} (\text{mod } 2) \quad \text{and} \quad M_{240} \cong -M_{240} (\text{mod } 2),$$

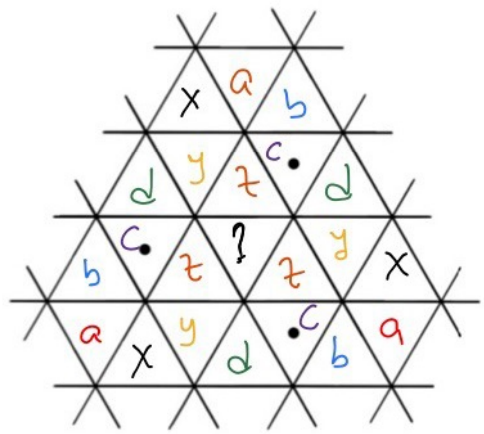
we can say

$$M = M + M_{120} + M_{240} + M_{120} + M_{240}$$

From here, it can be seen that $M + M_{120} + M_{240} \in S$, and $\quad M_{120} + M_{240} \in K$.

However, we have shown that ϕ is a bijective transformation. Since ϕ is a one-to-one and onto transformation, $|D| = |S||K|$. Then, when we find the number of elements of the set S, we find the number of sequences in $|K| = x$ where the sum of the numbers between both parallel lines is even.

To find $|S|$, let's write the numbers a, b, c, d arbitrarily in a symmetrical arrangement so that they fit into the triangles in the figure.



It can be seen that the remaining x, y, z numbers can be selected appropriately and uniquely so that the sums between each parallel line are even and the sequences are symmetrical. The number x in the triangle should be 0, if $a + b$ is even, 1 if $a + b$ is odd. So x depends on a and b.

Thus, we have scanned all the triangles in the sequence. And we made 4 arbitrary selection when making the sequence. If we take into account that they can swap places among themselves, we get $|S| = 2^4 = 16$.

So we obtain $|D| = 16|K| = 16x$.