## SOLUTION-APRIL 2022 MATH QUESTION–IZTECH

**Question.** Let $A = \{1, 2, \ldots, 9\}, B = \{11, 12, \ldots, 22\}$ and $C = \{24, 25, \ldots, 34\}$. Find the number of three element subsets of $A \cup B \cup C$ which do not contain consecutive integers.

**Solution.** If $\Gamma = \{1, 2, \ldots, n\}$, then the number of $k$ element subsets of $\Gamma$ which do not contain consecutive integers is given by $\binom{n-k+1}{k}$.

Let $A = \{1, 2, \ldots, 9\}, B = \{11, 12, \ldots, 22\}$ and $C = \{24, 25, \ldots, 34\}$ and suppose $X \subseteq A \cup B \cup C$ have 3 elements such that no element of $X$ are consecutive. We need to examine the following 7 cases :

(1) If all elements of $X$ belong to $A$, then we have $\binom{7}{3} = 35$ subsets.
(2) If all elements of $X$ belong to $B$, then we have $B$ is $\binom{10}{3} = 120$ subsets.
(3) If all elements of $X$ belong to $C$, then we have $\binom{9}{3} = 84$ subsets.
(4) If 2 elements of $X$ belong to $A$, and one element of $X$ belongs to $B$ or $C$, then we have $\binom{9-2+1}{2}(12 + 11) = 644$ subsets.
(5) If 2 elements of $X$ belong to $B$, and one element of $X$ belongs to $A$ or $C$, then we have $\binom{12-2+1}{2}(9 + 11) = 1100$ subsets.
(6) If 2 elements of $X$ belong to $C$, and one element of $X$ belongs to $A$ or $B$, then we have $\binom{11-2+1}{2}(9 + 12) = 945$ subsets.
(7) If one element of $X$ belongs to $A$, one element of $X$ belongs to $B$ and one element of $X$ belongs to $C$, then we have $\binom{9}{1}\binom{12}{1}\binom{11}{1} = 1188$ subsets.

Thus, the number of three element subsets of $A \cup B \cup C$ which do not contain consecutive integers is

$$35 + 120 + 84 + 644 + 1100 + 945 + 1188 = 4116.$$