AN APPLICATION OF GRAPHS: MATHEMATICS GENEALOGY PROJECT

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Purpose

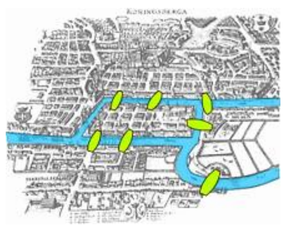
In this project, I study the Mathematics Genealogy Project and basic notions of Graph Theory to understand the structure of the Mathematics genealogy graph.

As an application I create an algorithm by using Pyhton programming language for creating IZTECH Mathematics Department's professors` genealogy trees.

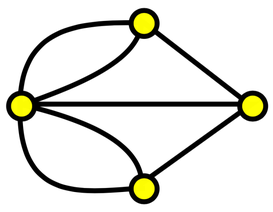
For a future work, I also investigate applications of graphs, specifically in Machine Learning.

What is Graph Theory?

Graph theory is the study of graphs that are mathematical structures, used to model pairwise relation between objects.



Königsberg Bridge Problem



Königsberg Bridge Problem as a Graph

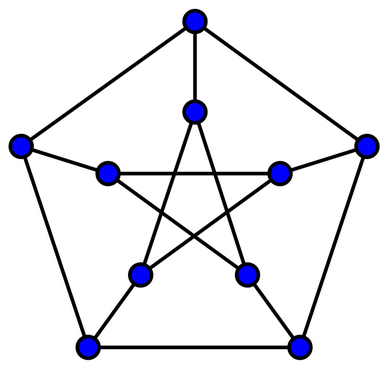
Graph theory was invented in 1736 by Swiss mathematician Leonhard Euler to solve Seven Bridges of Königsberg problem.

Königsberg Bridge Problem:

The challenge was to plan a walking route through the city that included crossing each of the seven bridges exactly once.

What is a Graph?

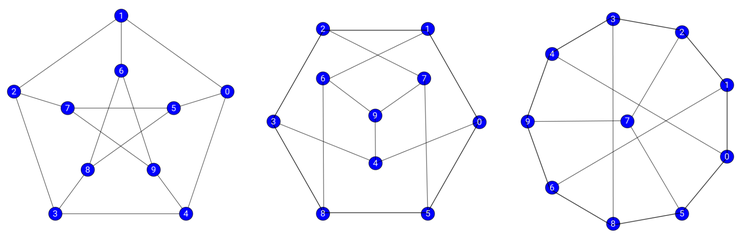
Definition: A *graph* G is an ordered pair (V(G), E(G)) consisting of a set V(G) of vertices and a set E(G) disjoint from V(G), of edges, together with an incidence function Ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.



Petersen graph: 10 vertices and 15 edges

Graph Isomorphism

Definition: Two graphs G_1 and G_2 are isomorphic if there exists a matching between their vertices so that two vertices are connected by an edge in G_1 if and only if corresponding vertices are connected by an edge in G_2.



Petersen graph transformation using isomorphism

Planarity

Definition: A graph G is *planar* if it can be drawn in the plane (R^2) in a way that no edges meet expect at a vertex with which they are both incident. Any such drawing is a plane drawing of G. A graph G is non-planar if no plane drawing of G exists.

Euler's Formula for planar graphs

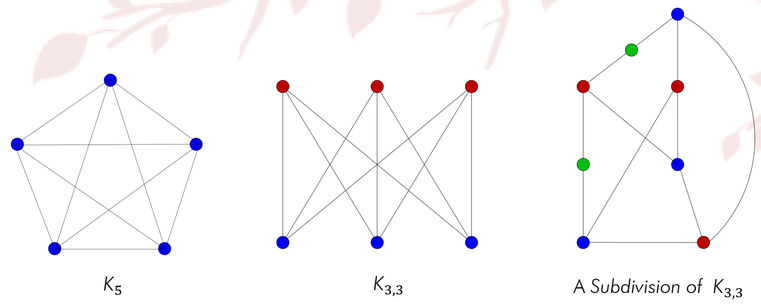
For any connected planar graph with *v* vertices, *e* edges and *f* faces, we have:

$$v - e + f = 2$$

Kuratowski's Theorem

Theorem: Let G be a graph. Then G is nonplanar if and only if G contains a subgraph that is a subdivision of either $K_{3,3}$ or K_5.

Definition: *Subdivision* is an operation that adds a vertex to an edge, dividing the edge into two.



K_5 $K_{3,3}$ A *Subdivision of* $K_{3,3}$

Mathematics Genealogy Project

Mathematics Genealogy Project founded in 1996 by Harry B. Coonce.

The project's purpose is to compile about ALL mathematicians of the world. They goal to list all individuals who have recieved a doctorate in mathematics.

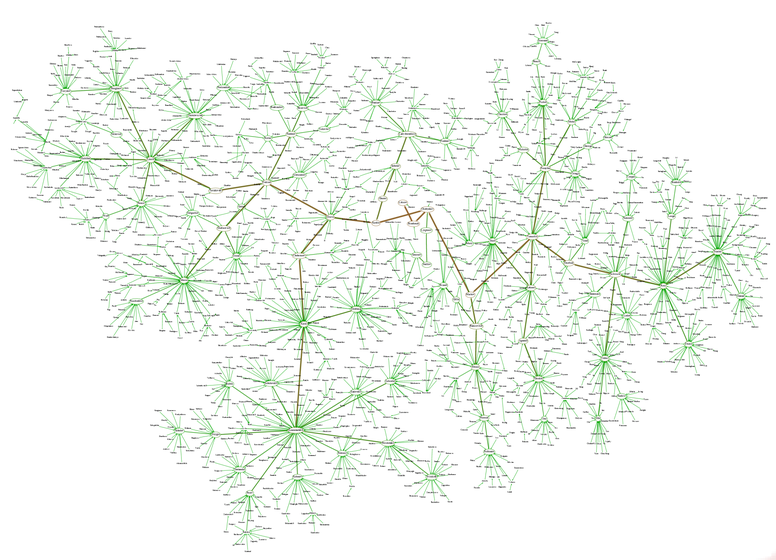
For each individual they show the following fields as data:

- The complete name of the degree recipient
- The name of the university which awarded the degree
- The year in which the degree was awarded
- The complete title of the dissertation
- The complete name(s) of the advisor(s)

Graph Structure of the Project

In this project the relation between mathematicians is created by using trees. Each advisor and student represents a vertex and if there exist a relation between them they are connected with edges.

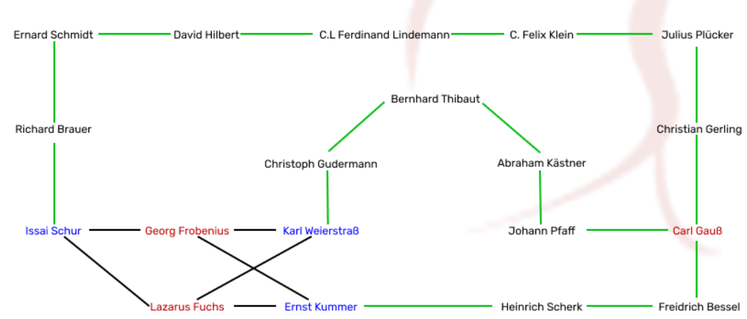
Data from 2016:
# of vertices =200,037
# of isolated vertices= 7639 (3.8%)
# of components of size two =1962 (advisor-advisee pairs where we have no information about the advisor). The largest component of the genealogy graph contained 180,094 vertices, accounting for 90% of all vertices in the graph. The main component has 7323 root vertices (individuals with no advisor) and 137,155 leaves (mathematicians with no students), accounting for 76.2% of the vertices in this component. The next largest component sizes were 81, 50, 47, 34, 34, 33, 31, 31, and 30.



An example of a tree in Mathematical Genealogy Project. Total 2130 mathematicians. Arrows point from a mathematician to his/her supervisor.

Project's Planarity

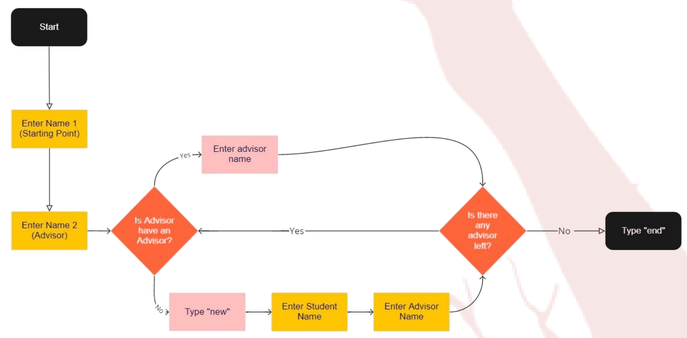
The Mathematics Genealogy Project graph is nonplanar. Professor Ezra Brown of Virginia Tech found the subdivision of $K_{3,3}$ as showed below.



C ---> Phyton

Mathematics Genealogy Project was written in C programming language that was developed in 1970s and has been very popular since then.

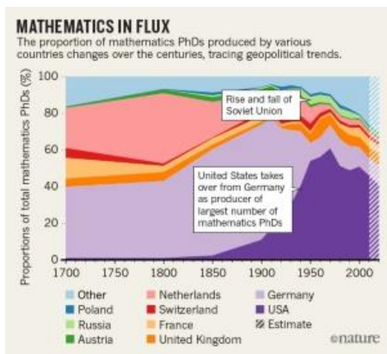
I have designed a Phyton algorithm for making the professors` mathematical genealogy tree in our department. I have used **networkx library** and a hand written algorithm. Here is wireframe of my algorithm:



Conclusions from the Mathematics Genealogy Graph

The "Mathematics Genealogy Project" offers us the opportunity to examine things from many perspectives. According to researches most of the world's mathematicians belong to only 24 scientific 'families', one of which dates back to the fifteenth century.

According to the structure of the Middle Ages, while scientific studies were intense in Europe at that time, when we came to the 1200s, we can see that mathematicians in Europe traveled to the East to receive education and that the real origin came from the East.

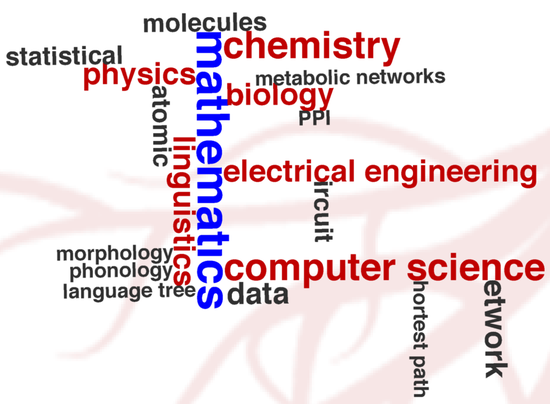


In the 1900s, with the rise of America, you can see that the majority of education shifted to America.

As a conclusion we can say that Mathematicians evolving depends to historical and political events.

Applications of Graph Theory

Graph Theory is used in vast area of science and technologies. Since we can represent every relation between objects we can use graph theory in almost everything.



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