

JANUARY 2022 MATH QUESTION–IZTECH

Let A be the subset of positive integers whose members are coprime to 6. To illustrate,

$$A = \{1, 5, 7, 11, 13, 17, 19, 23, 25, \dots\}.$$

Consider the sequence of sums of reciprocals of the members of A , for instance

$$a_1 = 1, a_2 = 1 + \frac{1}{5}, a_3 = 1 + \frac{1}{5} + \frac{1}{7}, \dots,$$

$$a_9 = 1 + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \frac{1}{17} + \frac{1}{19} + \frac{1}{23} + \frac{1}{25}.$$

In other words, a_n denotes the sum of reciprocals of the first n members of A . Prove that a_n is not an integer if $n > 1$.